

# Technical Comments

## Comment on "Vortex Velocity Distributions at Large Downstream Distances"

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In a recent Technical Note, Logan<sup>1</sup> presented axial and tangential velocity profiles measured in a wing trailing vortex. The Note made no mention of the mechanism used to obtain these velocities except to state that a 5-hole pressure probe was used and velocities were calculated from the experimentally obtained pressures. This procedure, using a simple 5-hole

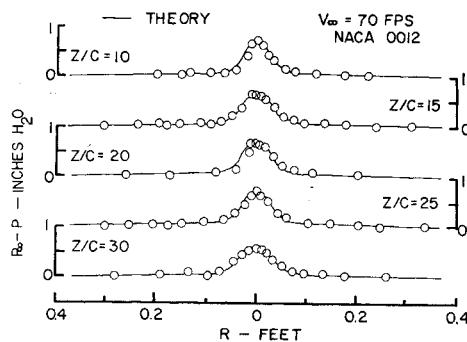


Fig. 1 Static pressures across vortex.

yawhead probe to give flow total pressure and angularity, is a common one and is quite accurate provided the local static pressures are known. However, it is apparent that Logan, as many before him, has assumed that static pressure is a constant through the vortex and has used this assumption to compute his component velocities. This rather common assumption is an intuitively erroneous one and has contributed greatly to the lack of understanding of trailing vortex structures that has existed over the years.

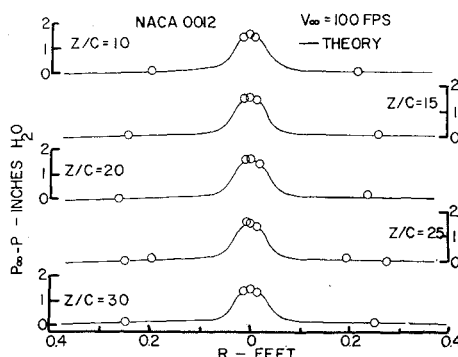


Fig. 2 Static pressures across vortex.

Past investigations of trailing vortices performed using hot wire anemometers<sup>2,3</sup> have revealed a velocity overshoot in the vortex core while pressure probe studies (assuming constant static pressure) have shown a velocity deficit such as that given by Logan. The discrepancy is an obvious one, but one which has gone unresolved until recently when Mason<sup>4</sup> conducted his trailing vortex studies using the method developed by Winternitz<sup>5</sup> to obtain static pressures from the yawhead probe. Using Winternitz method, Mason found static pressure variations across the vortex from an NACA 0012 wing at 7-1/2 degrees angle of attack as shown in Figs. 1 and 2. Figure 1 shows the pressure variation across the vortex at distances from 10 to 30 chordlengths downstream for a freestream speed of 70 FPS, Fig. 2 shows the results at 100 FPS. Both figures show both the experimental data and the theoretical distribution based on a simple Rankine vortex model. Failure to account for this pressure variation results in serious errors in some of the final velocity data.

Figure 3 illustrates the problem quite well. If Mason's pressure probe data is reduced to give the axial velocity in the

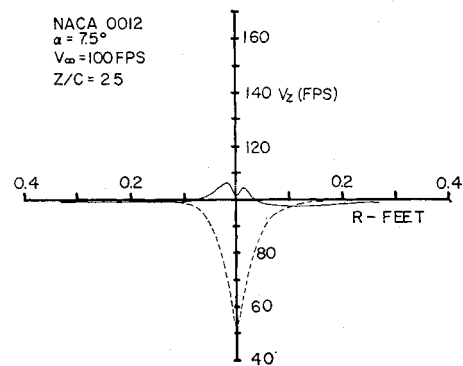


Fig. 3 Axial velocity distributions.

vortex while assuming constant static pressure, the curve that results is very similar to those presented by Logan. If, however, the static pressure variation is taken into account a quite different result is obtained which is more like the data previously obtained from hot wire measurements. The resulting error is most pronounced in the axial velocity case but it also has a slight effect on the tangential velocities as shown in Fig. 4.

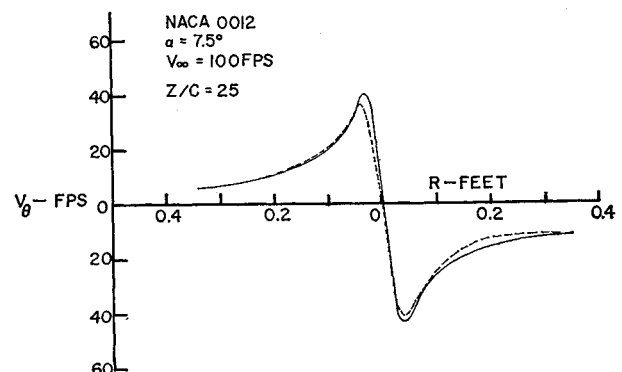


Fig. 4 Tangential velocity distributions.

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The errors which have resulted from the constant static pressure assumption in the past are one very probable reason for the past inability to achieve any kind of satisfactory correlation between flight test data and wind tunnel data and have undoubtedly slowed progress in obtaining a satisfactory theory for predicting the wake turbulence hazard.

#### References

- <sup>1</sup> Logan, A. H., "Vortex Velocity Distributions at Large Downstream Distances," *Journal of Aircraft*, Vol. 8, No. 11, Nov. 1971, pp. 930-932.
- <sup>2</sup> Fage, A. and Simmons, L. F. G., "An Investigation of the Air-Flow Pattern in the Wake of an Aerofoil of Finite Span," *Philosophical Transactions of the Royal Society*, A225, 1925, pp. 303.
- <sup>3</sup> Chigier, N. A. and Corsiglia, V. R., "Tip Vortices-Velocity Distributions," Presented at the 27th Annual National V/STOL Forum of the American Helicopter Society, Preprint 552, Washington, D. C., May 1971.
- <sup>4</sup> Mason, W. H., "Farfield Structure of an Aircraft Trailing Vortex, Including Effects of Mass Injection," Masters thesis, Nov. 1971, Virginia Polytechnic Institute and State University Aerospace Engineering Department, Blacksburg, Va.
- <sup>5</sup> Winternitz, F. A. L., "Probe Measurements in Three-Dimensional Flow," *Aircraft Engineering*, August 1956, pp. 273.

## Reply by Author to J. F. Marchman

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#### Nomenclature

- $C_{PP}$  = pressure coefficient of a side port  
 $P$  = pressure at a side port  
 $P_S$  = static pressure  
 $V$  = resultant velocity  
 $\rho$  = density

MARCHMAN is incorrect in his statement that I assumed that the static pressure is constant through the vortex. My data were reduced in the following manner. The static pressure probe was calibrated by first considering the pressure coefficient at one of the side ports while the probe was in a uniform stream which is what the probe sees when aligned with the resultant velocity inside the vortex. We can express the pressure coefficient at a side port as

$$C_{PP} = (P - P_S) / \frac{1}{2} \rho V^2$$

The pressure difference between the stagnation point and the side port can also be expressed as

$$P_S + \frac{1}{2} \rho V^2 - P = \Delta P$$

Combining these two expressions results in

$$\frac{1}{2} \rho V^2 (1 - C_{PP}) = \Delta P$$

$$V = (2\Delta P / [\rho(1 - C_{PP})])^{1/2}$$

where  $C_{PP}$  will be constant for the probe and position of the ports. As the procedure shows there was no assumption that the static pressure was constant.

The second value needed for the derivation of the axial and tangential velocities is the angularity of the resultant flow. The angularity was determined by independently measuring

the inclination of the pressure probe once it was aligned with the resultant velocity inside the vortex. The inclination of the probe and the magnitude of the resultant velocity were then used to derive the axial and tangential velocities. A complete description of the probe and the experimental method can be found in Ref. 1.

#### References

- <sup>1</sup> Logan, A. H., "A Solution to the Vortex Breakdown Phenomenon in a Trailing Line Vortex," M. S. thesis, 1966, The Pennsylvania State Univ., University Park, Pa.

## Comments on "Convergence Proof of Discrete-Panel Wing Loading Theories"

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THE Note<sup>1</sup> by J. DeYoung is unusual in that no reference to any of the vast number of published papers by workers other than himself in the very active field of thin-wing loading appears. In general, this can hardly be regarded as in the best interests of scientific method; but in this case it happens to be particularly important because even a brief acquaintance with the current literature would have shown the author that the inversion of the loading matrix—whose elements are given (in the notation of Ref. 1) by:  $1/(2m - 2n + 1)$  is only a trivial part of the problem since the result is known.

In fact, it is quoted as an example problem in a textbook by Isaacson and Keller.<sup>2</sup>

Furthermore, through communication from W. P. Jones the writer subsequently found that this result dates (at least) back to the work of A. R. Collar<sup>3,4</sup> in 1950. That the essential result for the inversion was known was reported in Ref. 5 and has subsequently been widely circulated and quoted (e.g., Refs. 6-8).

#### References

- <sup>1</sup> DeYoung, J., "Convergence Proof of Discrete-Panel Wing Loading Theories," *Journal of Aircraft*, Vol. 8, No. 10, Oct. 1971, pp. 837-839.
- <sup>2</sup> Isaacson, E., Keller, H. B., *Analysis of Numerical Methods*, 1st ed., Wiley, New York, 1966, pp. 217-218.
- <sup>3</sup> Collar, A. R., "On the Reciprocal of a Segment of a Generalized Hilbert Matrix," *Proceedings of the Cambridge Philosophical Society*, Vol. 47, Pt. 1, 1950.
- <sup>4</sup> Collar, A. R., "On the Accuracy of the Representation of a Lifting Line by a Finite Set of Horseshoe Vortices," *The Aeronautical Quarterly*, Vol. IX, Aug. 1958, pp. 232-250.
- <sup>5</sup> James, R. M., "On the Remarkable Accuracy of the Vortex Lattice Discretization in Thin-Wing Theory," Rept. DAC 67211, 1969, Douglas Aircraft Co., Long Beach, Calif.
- <sup>6</sup> Rodden, W. P., Giesing, J. P., and Kalman, T. P., "New Developments and Applications of the Subsonic Doublet-Lattice Method for Nonplanar Configurations," *AGARD Symposium on Unsteady Aerodynamics for Aero-elastic Analyses of Interfering Surfaces*, Paper 4. Ionsberg, Oslofjorden, Norway, 3-4 Nov. 1970, AGARD-CP-80-71.
- <sup>7</sup> Kalman, T. P., Rodden, W. P., and Giesing, J. P., "Application of the Doublet-Lattice Method to Nonplanar Configurations in Subsonic Flow," *Journal of Aircraft*, Vol. 8, No. 6, June 1971, pp. 406-413.
- <sup>8</sup> Stark, V. J. E., "A Generalized Quadrature Formula for Cauchy Integrals," *AIAA Journal*, Vol. 9, No. 9, Sept. 1971, p. 1854.

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